The Kinetic Theory of a Gas Constituted of Spherically Symmetrical Molecules.

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(Abstract.)

In order to extend Maxwell's later method of developing the dynamical theory of a gas* to cases other than that which he considered (viz., a gas whose molecules are point centres of repulsive force varying inversely as the fifth power of the distance), a knowledge of the velocity distribution function, in the disturbed state of the gas, is necessary. In this paper the simplest possible form is assumed for the function, consistent with the fulfilment of certain preliminary conditions. This form is

$$\left(\frac{hm}{\pi}\right)^{\frac{3}{2}} e^{-hm\Sigma(u-u_0)^2}\left\{1+\mathrm{F}(u-u_0, v-v_0, w-w_0)\right\},$$

where F is a polynomial, in the three variables indicated, of the third degree. The theory of viscosity and thermal conduction, in simple and mixed gases, is developed without assuming any property of the molecules beyond that of spherical symmetry.

Perhaps the most interesting result is the relation between the viscosity μ, the thermal conductivity ϑ, and the specific heat at constant volume, C_v, for a simple monatomic gas, viz.,

$$\vartheta = \tfrac{5}{2}\mu C_v.$$

Maxwell obtained this relation for a gas of the particular kind above mentioned, but it is here obtained without reference to the law of action between the molecules. Schwarze's experiments† on argon and helium show that the law is very closely followed in the case of these two gases (the only monatomic gases for which data are available); Meyer's formula, $\vartheta = 1{\cdot}60\mu C_v$, is widely departed from.

Others of the formulæ obtained are unmanageable in their general form, but have been worked out completely in the case of three special laws of action between the molecules; these are, that the molecules behave as (*a*) elastic spheres, (*b*) elastic spheres which attract one another, or (*c*) point

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* 'Phil. Trans.,' 1867 and 1879; or 'Scientific Papers,' vol. 2, pp. 23, 681.

† 'Ann. d. Phys.,' 1903, vol. 11, p. 303; 'Phys. Zeitschrift,' 1903, vol. 4, p. 229.

centres of repulsive or attractive force varying inversely as the *n*th power of the distance.

The assumed form of the velocity distribution function, however, is incomplete. Enskog* has shown that the function F must take the form of an infinite series, except in Maxwell's special case. On this account the numerical constants of the formulæ obtained may require to be modified by factors which in cases (*a*) and (*c*) are numerical only, and in other cases also depend on the temperature. The determination of these factors is extremely difficult, and is not effected in the paper; but the comparison of the formulæ, as they stand, with the experimental results appears to indicate that the errors of approximation are small.

Maxwell's theory of diffusion is also extended to the most general kind of monatomic gas; this part of the work is not affected by the assumption concerning F, which is not required in this part of the theory.

The comparison of the formulæ with experimental results tends to show that the second type of molecule considered, viz., an elastic sphere surrounded by a field of attractive force, gives, on the whole, the best representation of the behaviour of actual molecules, so far as concerns the three particular phenomena of gases here discussed.

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* 'Phys. Zeitschrift,' 1911, vol. 12, p. 58.